Crop Circles
Teacher Notes

Introduction

Crop circles have been around for a long time – there is a woodcut of a 'Mowing Devil' in a pamphlet produced in Hertfordshire in 1678 - but interest became more intense in the late 1970s following the work of two self confessed hoaxers Messrs Bower and Chorley.

Crop circles appear annually, and as such have to be accepted as a 'real' phenomenon, whatever their cause. There is scope to research the background as well as discuss the morality of the deliberate damage to crops by those who confess to creating patterns!

In investigating crop circles there are obvious opportunities for geometric construction both with software and with compass and ruler: many shapes are based on the circle, as well as regular polygons. The investigation of symmetries is also intrinsic to the study of crop circle patterns. Websites with detailed photographs and commentaries include :

www.**cropcircle**connector.com   www.uk**cropcircles**.co.uk
www.**cropcircle**research.com   www.temporarytemples.co.uk

This activity offers opportunities to explore symmetry and to name shapes, but then develops a more integrated approach involving area calculations, and calculating proportions and ratios. Topics covered include:

- Recognise and name different polygons
- Identify reflection and rotational symmetry
- Calculate areas
- Calculating proportions and ratios
- Investigate a sequence

Resources

The TI-Nspire documents *CompassRose.tns*, *Fractal.tns* and *HouseofPi.tns* are needed for this activity. A student handout guides students in the use of the TI-Nspire documents

TI-Nspire skills students will need

Transferring a document to the handheld
Opening a document on the handheld
Moving between pages of a document
Drawing line segments and polygons
Using the Measurement tool for lengths and areas

The activities

As a preamble to the topic, teachers may like to put together a slide show of crop circle photographs. There are several websites carrying appropriate images – see above. The symmetries of these shapes are an obvious starting point.

For the starter activity, use the document *CompassRose.tns* either as a whole class activity with the TI-Nspire Teacher software projected onto a screen or as an individual exercise, in which case it will need to be transferred to the handhelds.

The main activity is based on a fractal pattern, and encourages pupils to calculate the areas of triangles, and find a sequence of fractions of flattened areas, which can be linked to geometric sequences. An extension activity is included: this is based on a deceptively simple crop circle, called the House of Pi, which reveals some interesting area ratios.

Starter
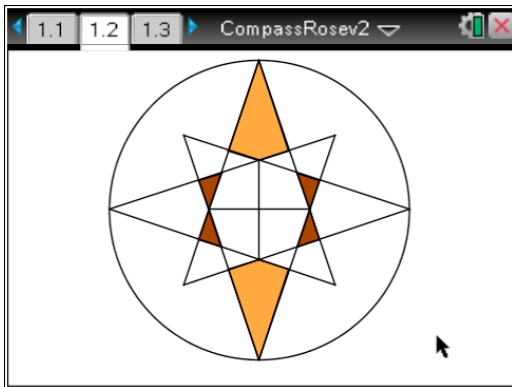
Students can draw in the lines of symmetry, identify the degree of rotational symmetry, and identify all the different polygons that they can see. It helps to focus on the different shapes if students draw over the polygons, using the polygon tool. Counting how many polygons of each type is also possible.
Answers can go on the second page.
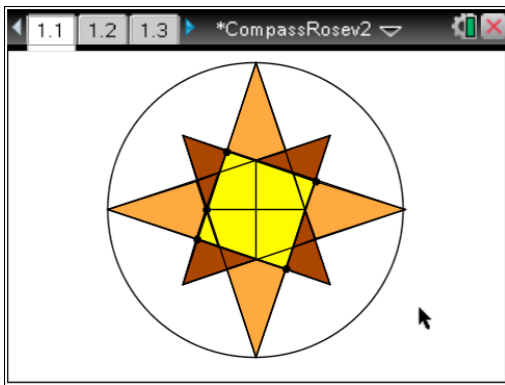
Students could also change colours or attributes. (Move over a polygon and press [ctrl] [menu] for the contextual menu).

Changing colours will change the type of symmetry – how many different types are there?
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Main Activity – Fractal Area
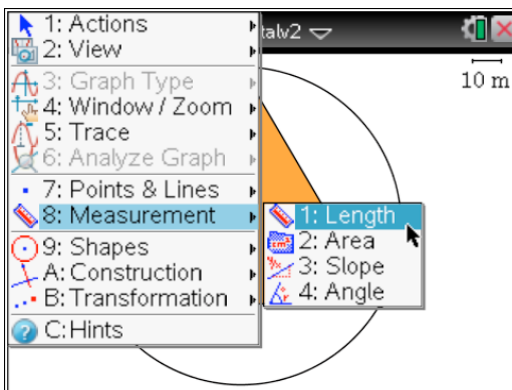
This activity involves constructing the so-called Sierpinsky Sieve, starting with an equilateral triangle.

Students can calculate the area of the triangle, using the menu **8: Measurement 1: Length** to find the base and height, and a calculator section on page 1.3 for the necessary calculation.

The area can be checked using the area option, also from the **8: Measurement** menu.
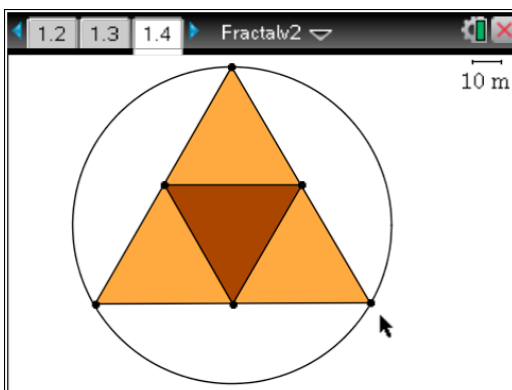
The triangle is then divided as shown, and the ratio of flattened corn (shaded area) to unflattened corn (unshaded area) found.

Questions are found on page 1.5:
a) what fraction of the triangle is flattened (shaded brown)?
b) What is the ratio of flattened to unflattened corn?
c) What is the area of each triangle?
d) What percentage of the crop has been damaged?
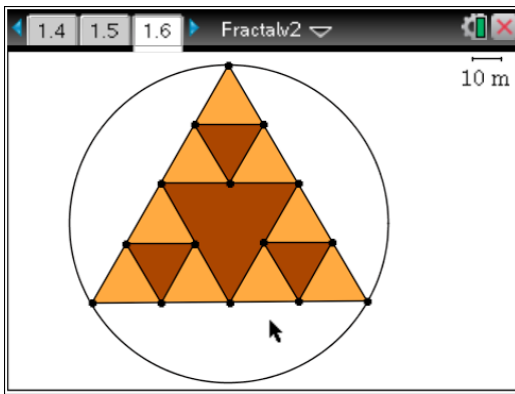
A further split is made, and the same questions are posed.

On page 1.8 students are asked to predict the next fraction, ratio, area and percentage when further iterations are constructed.
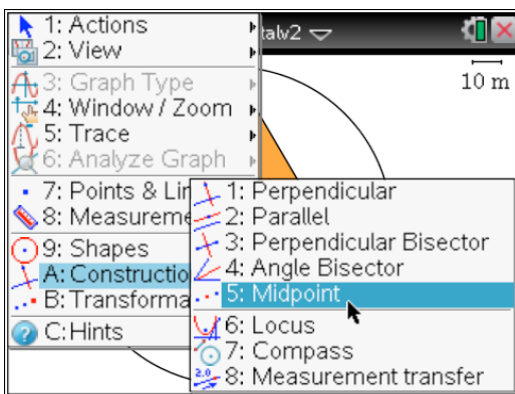
On page 1.9 students can check their predictions for this division by drawing.

They can use the Midpoint tool to divide the sides of the orange triangles: they will need to click on the two ends of sides to be divided.

Once the sides are split, triangles can be added. They should then use the Triangle tool and click on the vertices of the new triangles .
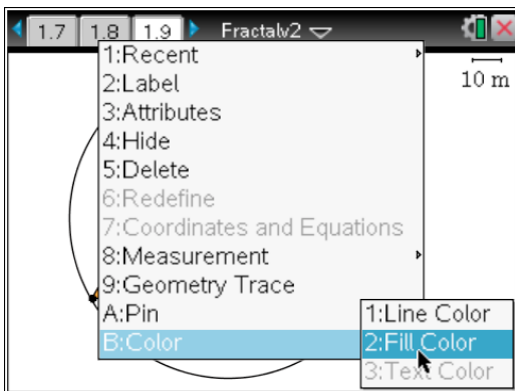
Finally they will want to colour the triangles representing newly flattened corn. To do this they should move the cursor to the triangle to be coloured and press ⌈ctrl⌉ ⌈menu⌉ for the contextual menu as shown here.

With the teacher software It is possible to speed up the colouring process by selecting multiple triangles (Shift click)
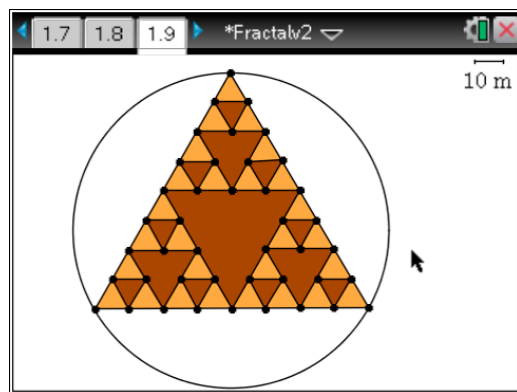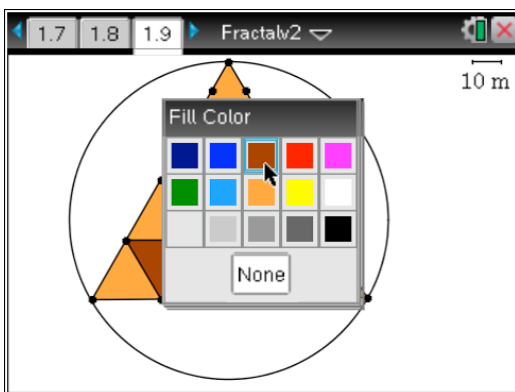
Extension Activity – the House of Pi
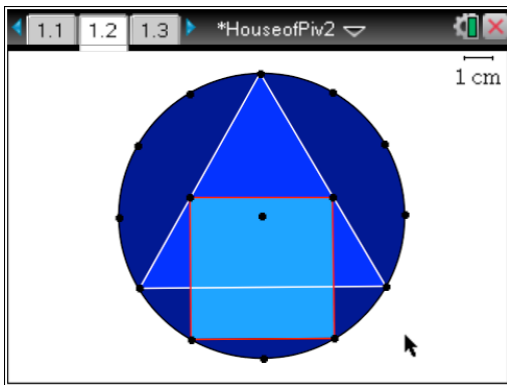
This delightfully simple looking crop circle is constructed by superimposing an equilateral triangle on the circumference of a circle.
The square is then constructed on the base of an implied hexagon, so that its side length is equal to the radius. This means that the area of the circle is πr^2 and the area of the square r^2.
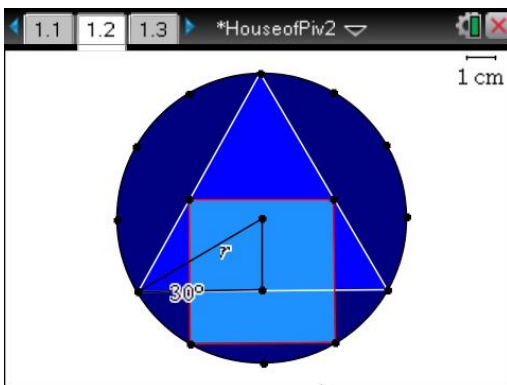
Students can measure the areas of the two triangles to find the scale factor linking them.

By constructing the triangle shown, using the centre of the circle for one vertex, which has an angle of 30°, it can be seen that the base of the larger triangle is 2 x $r\cos (30^{\circ})$ = $r\sqrt{3}$.

The two triangles are similar therefore the area scale factor is $(\sqrt{3})^2$ = 3.....

Background to the shapes:

The Compass Rose: this design appeared in 1996. It is constructed by marking the vertices and midpoints of the sides of a square, and then joining up every third point. This makes for four 3-4-5 triangles, as well as two smaller squares appearing at the centre. The ratio of the smaller squares to the large enveloping square is 1:5, whilst other ratios include 1:8 for the ratio of the smaller 8 triangles to the main triangles, and the ratios of the rhombi to the centre square is 3:2. These ratios can be checked by drawing polygons over the relevant sections and measuring their areas. Proof is left to the enthusiast!

The Sierpinsky Sieve: this is the simplest of the fractal forms which have appeared at regular intervals. The triangle is successively divided into four, with the central section left untouched at each stage. It appeared in Hampshire in June 1999.

The House of Pi: this elegantly simple design appeared in Old Sarum, Wiltshire, in July 1998. It can be constructed by inscribing a dodecahedron inside a circle. The triangle is formed by joining every fourth vertex, and the square is constructed using one side of the implied hexagon as its base. This leads to the simple ratio of areas explored in the extension activity.

References:
Crop Circles The Hidden Form by Nick Kollerstrom Wessex Books ISBN 1-903035-11-2 gives a fascinating insight into the geometric structures and properties of Crop Circles.