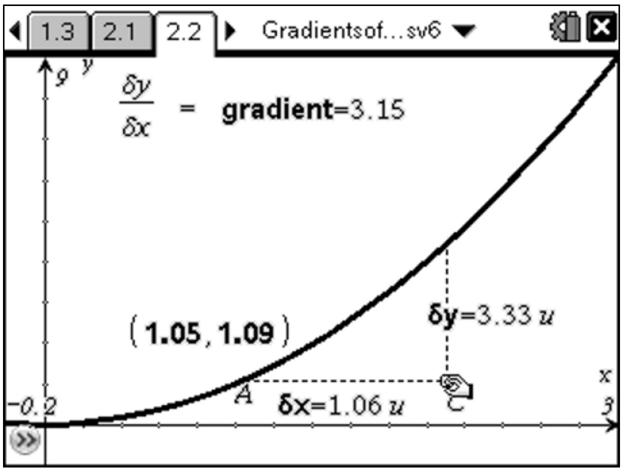
Exploring Gradients of Curves

Mathematical Content:	Technical TI-Nspire Skills:
Limits Differentiation	Manipulation of Geometric Constructions Data Capture

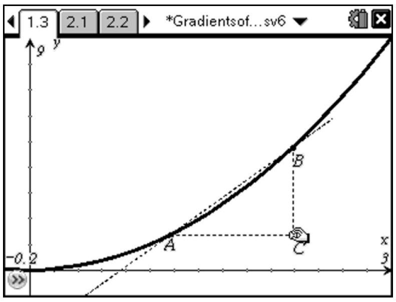
This activity provides an introduction to differentiation in the context of the gradients of curves. TI-Nspire allows students to see that, as a chord is shortened, its gradient gets close to the gradient of the curve itself. The notation δx, δy and $\delta x/\delta y$ is introduced and students are able to experience dynamically that, as δx tends to zero, so $\delta x/\delta y$ tends to the gradient of the curve at the point.

There is TI-Nspire document called GradientsofCurves.tns. This can either be used on a class set of TI-Nspire handhelds with students working individually or in small groups or it could be used with the TI-Nspire software and projected onto a screen for class discussion.

The activity has been developed by Andy Kemp with significant input by Barrie Galpin.



Exploring Gradients of Curves – Student Worksheet

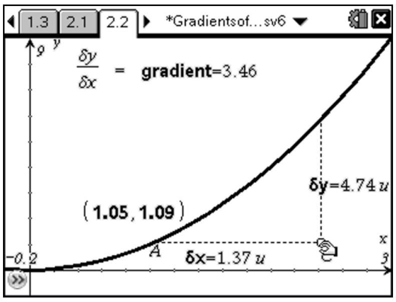


In this activity you will explore the limit of gradients of a chord and so discover the gradient of a curve at a point.

The activity begins with three introductory pages that give an overview of the task. Remember you can move between pages by pressing ctrl ▸.

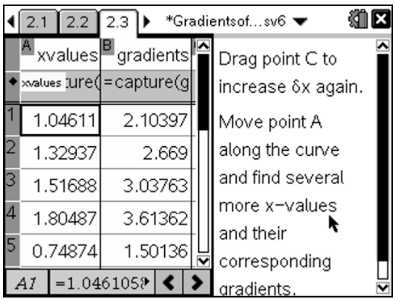
Task 1:

Start by reading the instructions on page 2.1. These explain how to manipulate the construction on page 2.2.



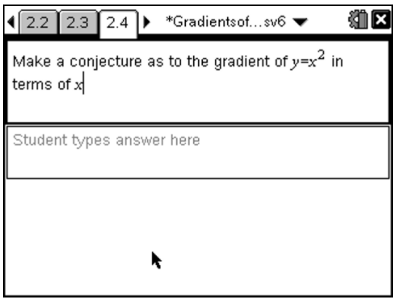
In this construction notice the labels δy and δx, which are used to represent a small change in y and a small change in x respectively. Also notice that the gradient $\delta y/\delta x$ is calculated. By making δx very small an approximation to the gradient of the curve $y=x^2$ at the point A can be found

Reduce the size of the chord, and observe how its gradient gets closer to the gradient of the curve at point A. When you have the chord sufficiently small you can record the x-value and gradient by pressing ctrl .



Repeat this process for a number of different values of x values: you can adjust the x-value by picking up and dragging point *A*. Recording the x-value and gradient at each point.

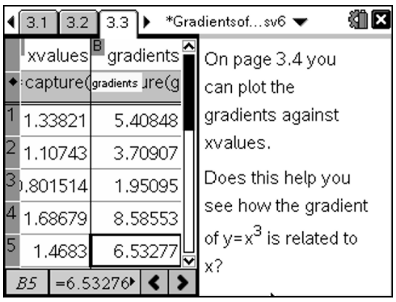
When you have recorded values for least 4 or 5 points have a look at the table on page 2.3, and try and form a conjecture as to the relationship between the x-value and the gradient of x^2 at that point.



When you have formed your conjecture enter it on page 2.4. Then check you answer with the suggested response by pressing menu and selecting "Check Answer".

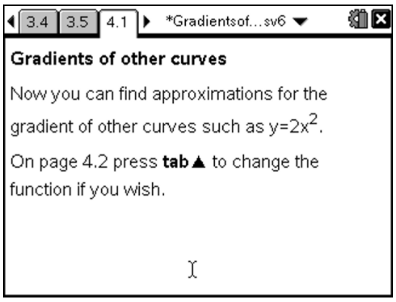
Task 2:

In this second task you need to repeat the process for the curve $y=x^3$.



When you have recorded values for five or six points on page 3.2 in the same way as you did above, take a look at the table on page 3.3. This time the relationship is a little more complicated and it may not be obvious at first glance what the gradient function is. On page 3.4 the x-values and their related gradients have been plotted on a graph. Have a look at this and see if it helps you identify the relationship.

Task 3:



In this third task you are asked to explore the gradient of $y=2x^2$ and then try to making a general conjecture about the gradient of $y=ax^n$. Again record values for five or six points on page 4.2. Use the table of values on page 4.3 and the associated graph on page 4.4 to try and deduce the gradient function for this example.

You may like to look at carrying out a regression on the data by going to page 4.4 and choosing from the Analyse menu option 6, Regression, then choose an appropriate regression: Quadratic, Cubic, Quartic or Power. (Press (menu)(4)(6) and choose (4),(5),(6), or (7).)

If you want to explore further examples you can return to page 4.2 and change the function. Do this by bringing up the input line (press (tab)▲) and editing the function. You will also need to clear the values already recorded by going to page 4.3 and clearing each column by moving to the head of the column and choosing from the Data menu option 4, Clear Data (Press (menu)(3)(4)).

When you feel you can describe the gradient function for $y=2x^2$ and $y=ax^n$, move on to page 4.5 and describe the gradient functions. When you are confident in your answer, you can check it by again pressing (menu) and choosing “Check Answer”.

Task 4:

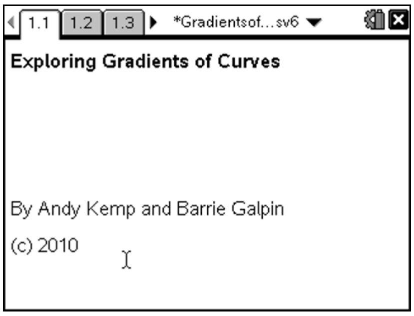
This final section consists of some notes pages that present an algebraic approach called ‘differentiation from first principles’. It outlines a proof of the result you discovered in the first task. At the end of it you are encouraged to use this as a model to carry out the process of differentiation from first principles for $y=x^3$.

Gradient of $y=x^3$ *from first principles:*

Exploring Gradients of Curves – Detailed notes

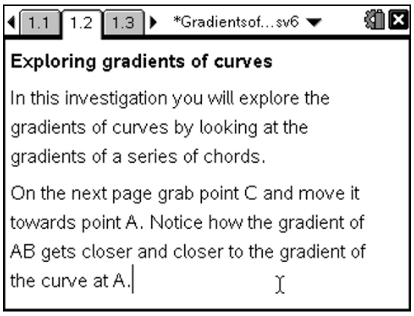
These notes briefly describe the content of each page and draw attention to any important elements.

Page 1.1



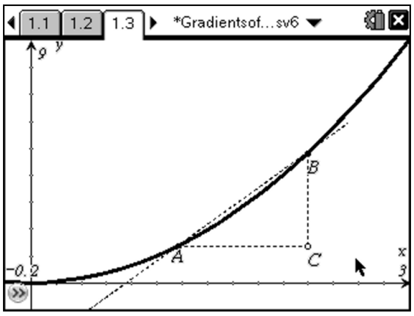
This is the title page.

Page 1.2



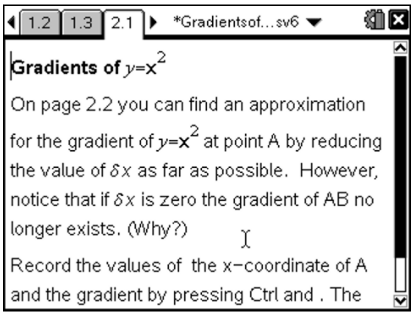
This page introduces the general concept of the task and outlines what the students should do on the following page.

Page 1.3



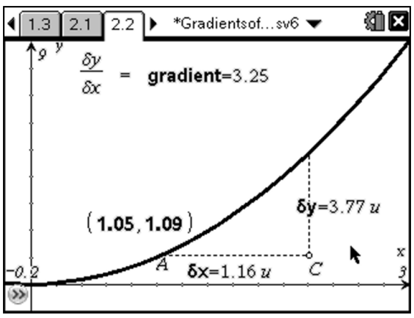
On this page students are able to drag the point C towards A, observing that the slope of the chord gets closer to the gradient of the curve as the length AC gets smaller.

Page 2.1



This page introduces and explains the task and outlines how students can record data.

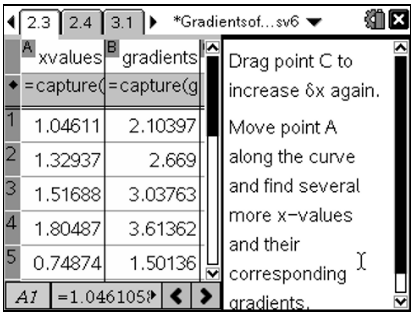
Page 2.2



Here students are again able to adjust the size of the chord by dragging point C (the open circle). They can also observe how the gradient of the chord gets closer to the gradient of the curve. When they have the chord sufficiently small they can record the x-value and gradient by pressing ctrl . .

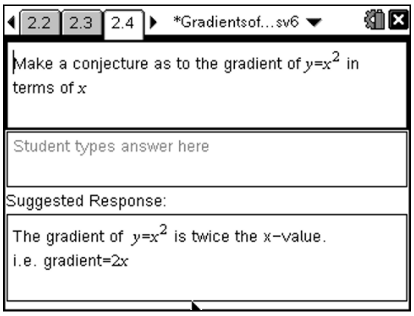
This should be repeated with point A in five or six different positions.

Page 2.3



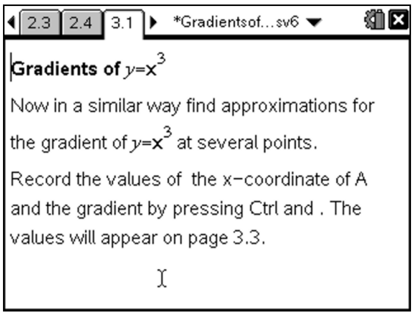
This page contains the captured data. Students should notice that in each case the gradient is twice the x-value.

Page 2.4



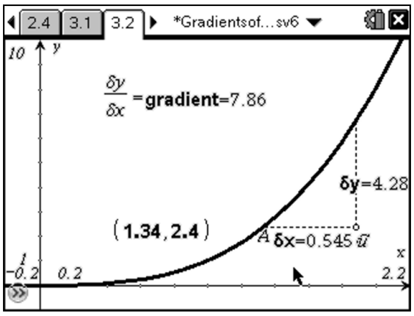
On this page students should try to summarise their findings by giving a formula for the gradient. When they have entered their response they are able to compare it with the suggested answer by pressing (menu) then selecting "Check Answer"

Page 3.1



This page introduces the second task, which this time look at the $y=x^3$ function.

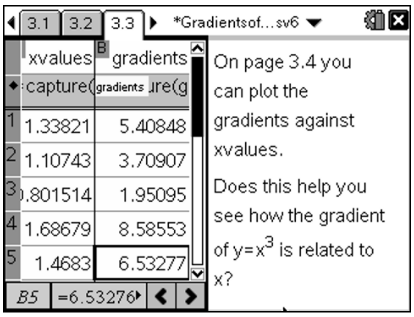
Page 3.2



As before, students can reduce the size of the chord and observe how the gradient of the chord gets closer to the gradient of the curve. They can record the x-value and gradient on page 3.3 by pressing (ctrl)(.)

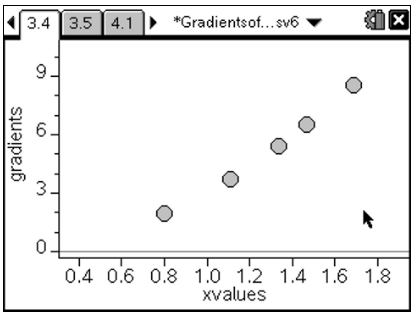
This should be repeated for five or six x-values.

Page 3.3



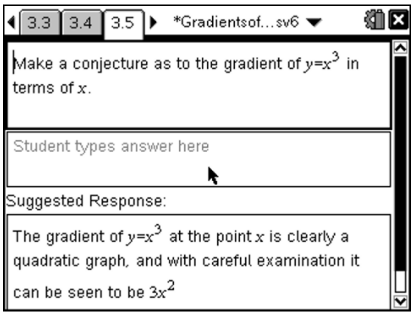
This page contains the captured data.

Page 3.4



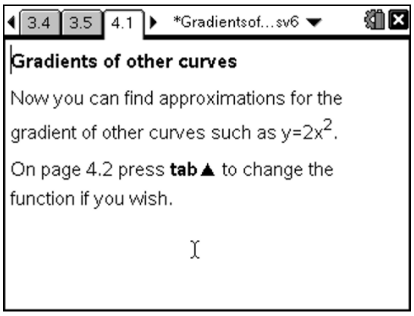
This page automatically plots the points captured on page 3.3. Students should recognise the shape of the graph and use it to help identify the relationship.
If they are struggling then they can perform a quadratic regression on the data by pressing (menu)(4)(6)(4).

Page 3.5



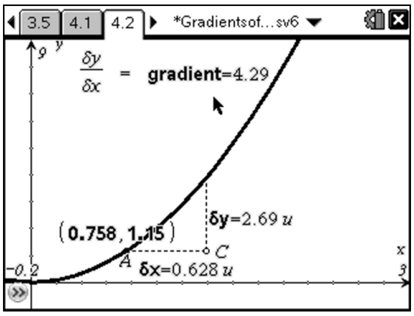
Another page where students can summarise their findings and check their answers.

Page 4.1



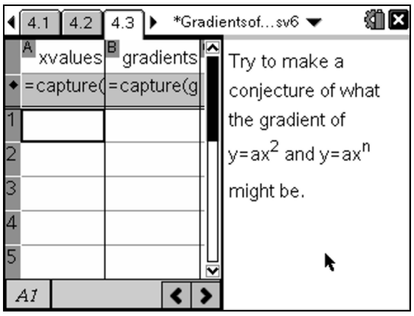
The third task is to look at the $y=2x^2$ function.

Page 4.2



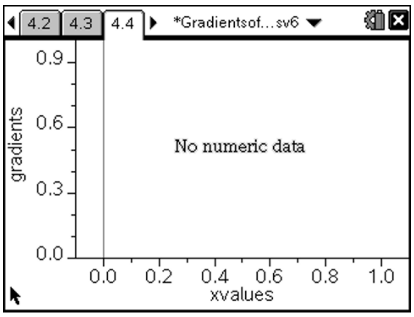
Once again the chord can be reduced until the gradient is close to the gradient of the curve. The x-value and gradient should be recorded for five or six different positions of A.

Page 4.3



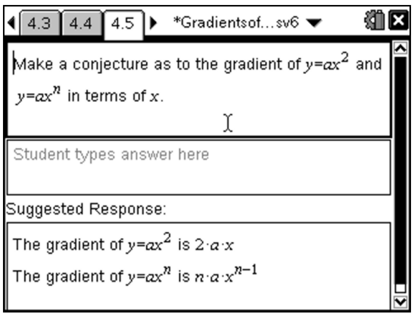
This page contains the captured data.

Page 4.4



Points are plotted enabling students to identify the relationship between the x-value and the gradient. If they are struggling then they can try performing various regressions on the data by pressing menu 4 6

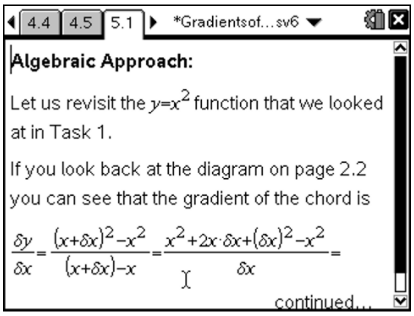
Page 4.5



Another page for summarising findings.

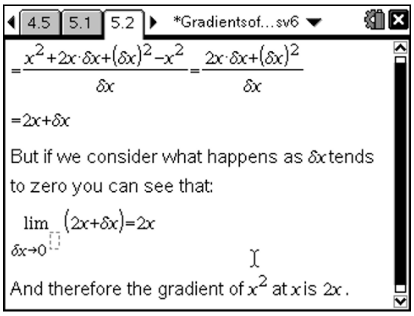
Students may wish to try another function and to explore further examples. If so they need to return to page 4.2 and change the function by bringing up the input line (press (tab) ▲) and editing the function **f1**. They will also need to clear the values previously recorded on page 4.3: clear each column by moving to the head of the column and choosing from the Data menu option 4, Clear Data (Press (menu)(3)(4).)

Page 5.1



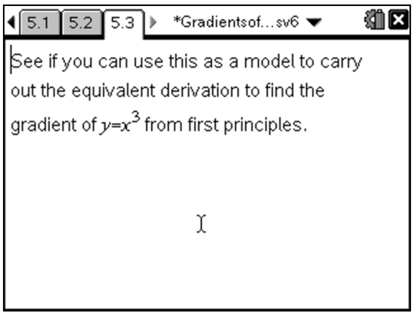
This page outlines the standard algebraic approach to finding the gradient function for $y=x^2$ from first principles.
The notes go through the process of expressing the gradient in terms of δx and δy which is then rewritten just in terms of δx, and then expanded.

Page 5.2



The derivation of the gradient function is continued, cancelling it down to $2x+\delta x$ and then looking at the limit as δx tends to zero, giving the gradient as $2x$.

Page 5.3



This final page encourages students to try and use the proof shown on pages 5.1 and 5.2 as a model to do the same for $y=x^3$.

The proof of the derivative of x^3 from first principles:

$$\frac{\delta y}{\delta x} = \frac{(x+\delta x)^3 - x^3}{(x+\delta x) - x} = \frac{x^3 + 3x^2\delta x + 3x(\delta x)^2 + (\delta x)^3}{\delta x} = \frac{3x^2\delta x + 3x(\delta x)^2}{\delta x}$$

$$= 3x^2 + 3x\delta x$$

Taking the limit gives:

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 3x^2$$