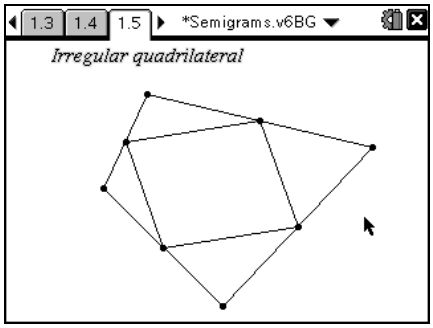
Joining midpoints to form semigrams

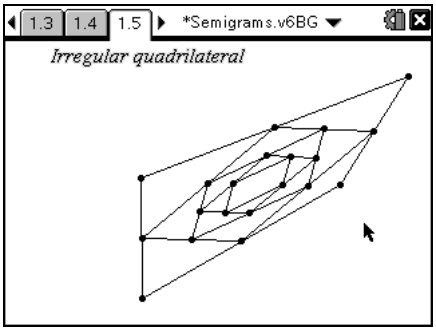
Teacher Notes

Introduction

What shape do you get when you join the mid-points of adjacent sides of a quadrilateral?
Can you explain why this is or prove your results?
What happens if you repeat the process one or more times?
In this activity students are able to carry out this investigation on Geometry pages on their TI-Nspire.



Semigram of an irregular quadrilateral



Semigram of semigram of…

What is a semigram?

This activity has been developed from an initial idea by Linda Tetlow an Education Consultant from Brighton who introduced it to her classes as an open-ended investigation after she found this question in *'Patterns in mathematics'* by Donald Eperson (Basil Blackwell, 1988).

> **A semigram is formed by joining the mid-points of consecutive pairs of sides any quadrilateral. Why is its area half that of the quadrilateral?**
>
> **A demi-semigram is the semigram of a semigram. Draw the semigrams and demi-semigrams of the given shapes (there were 10 different quadrilaterals) and complete the following table…** (there were headings such as '4 equal sides'; 'no parallel sides'; 'equal diagonals' etc)'.

Resources

There is a TI-Nspire document entitled Semigrams. This has six pages including five ready-drawn figures: a rectangle, kite, parallelogram, irregular quadrilateral and regular pentagon. Alternatively, students may start with a blank document and construct shapes for themselves.

Student notes (below) give guidance on use of the TI-Nspire handheld to construct a semigram.

Prior knowledge

To use the Semigrams document students will need to be able to: -

- open a TI-Nspire document and move between the pages of the document;
- move the cursor around the screen;
- grab and drag points on the screen;
- have some knowledge and understanding about the names and properties of quadrilaterals.

If starting with a blank TI-Nspire document students will need to use a wider range of Nspire tools from the Geometry menus.

Rationale for doing this activity

In the table below two types of activity are suggested depending on the experience of the students both with mathematics and with the technology.

- Activities in standard type use the diagrams given in the tns file.
- *Activities in italics require students to construct shapes themselves. This requires deeper mathematical understanding as well as greater familiarity with TI-Nspire.*

Objective	Further details
Opportunity to develop key concepts such as creativity and key processes such as analysing	For example: **Creativity**: Use existing knowledge to create solutions to unfamiliar problems; pose questions and develop convincing arguments. **Analysing**: Make connections within mathematics; visualise and work with dynamic images; make and begin to justify conjectures and generalisations, consider special cases and counter examples; engage with someone else's mathematical reasoning.
Introduce the features of the TI-Nspire Geometry application with little technical expertise required.	Using the given diagrams in the tns file students only need to use • Mid Point (Option 5 in the Constructions menu); • Segment (Option 5 in the Points & Lines menu). *Students could alternatively construct diagrams of their own using just a few more construction tools such as Point, Point On, Line, Parallel, Perpendicular, Hide/Show.*
Opportunity to use existing knowledge and to increase familiarity with quadrilaterals and their properties.	Make and justify conjectures about the shapes and properties of the semigrams drawn within given quadrilaterals.
Opportunity to use existing knowledge of properties of quadrilaterals and of parallel and perpendicular lines to construct quadrilaterals.	*Plan how to construct specific quadrilaterals so that their shape remains defined when they are distorted by dragging one or more points*
Opportunity to make connections between properties of shapes, parallel lines, congruent and similar triangles, and symmetry to form convincing arguments.	Justify conjectures about the shape of semigrams and dem-isemigrams ... *Justify construction methods for quadrilaterals*
Opportunity for students to suggest ways to extend the problem	*Students could extend their conjectures to shapes other than quadrilaterals or look at other features such as 'What happens to the area? ''. What happens to the lengths of the sides?'*

Student Notes

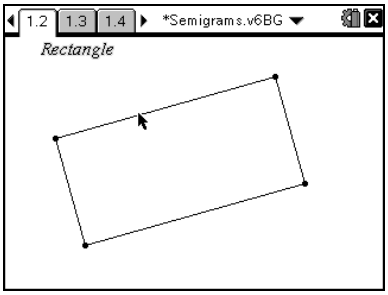
Investigating semigrams

The shape you get when you join the mid points of a quadrilateral is sometimes called a semigram. What is special about it?
Can you explain why this is or prove your results?
What happens if you repeat the process one or more times?

Open the TI-Nspire document called Semigrams.

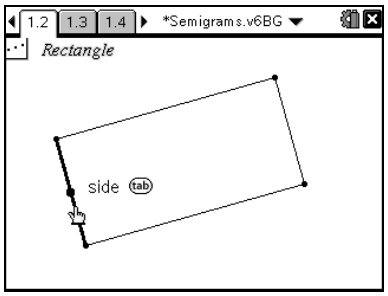
1) Start with a rectangle

On page 1.2 you will see a rectangle. Grab one of the corners and move it around. Will it always b a rectangle?



The first step is to mark the mid points of each side.

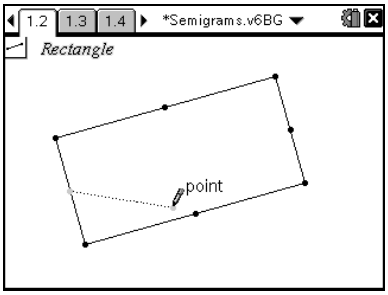
Press menu 9 5 to choose the Mid Point tool.
Move over each side and mark the midpoints by pressing enter or ⬚.



Next, join up the mid points with line segments.

Choose the Segment tool by pressing menu 6 5.

Move the cursor over the one of the midpoints and press enter or ⬚.
As you move the cursor a dotted line appears.



Move to the second midpoint and click ⬚ on it to fix the line segment.

Repeat for all four pairs of mid points.

a) What happens if you drag the corners of the original rectangle?
Does the shape inside stay the same?

b) Now mark the midpoints of the sides of the semigram and join these with line segments to form a demi-semigram.
What shape do you get?
Why do you think this is?
Is there a connection between the lengths of the sides of the original rectangle and the lengths of the sides of the semigram and demi-semigram?

c). What do you think will happen if you keep repeating the process? Why?

Other quadrilaterals

On the next pages on the *Ti-Nspire* docuement you will find a number of other shapes that you can use to draw semigrams and demi-semigrams, etc. in the same way as you did for the rectangle.

Try to predict what will happen and test to see if you are correct.

Try to explain why this is. To do this you could insert a Notes page (press ctrl doc 6).

Can you think of any other shapes that you would like to try?

You might want to try to draw your own shapes. If you do you will need to think carefully about the properties that are special for that shape and the constructions that you will need to do.

Check your construction to make sure the shape retains these special properties when you drag any of its corners.